

# HIGHER-DERIVATIVE GRAVITATION IN BOSONIC AND SUPERSTRING THEORIES AND A NEW MECHANISM FOR SUPERSYMMETRY BREAKING

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ABSTRACT. A discussion of the number of degrees of freedom, and their dynamical properties, in higher derivative gravitational theories is presented. The complete non-linear sigma model for these degrees of freedom is exhibited using the method of auxiliary fields. As a by-product we present a consistent non-linear coupling of a spin-2 tensor to gravitation. It is shown that non-vanishing  $(C_{\mu\nu\alpha\beta})^2$  terms arise in  $N = 1$ ,  $D = 4$  superstring Lagrangians due to one-loop radiative corrections with light field internal lines. We discuss the general form of quadratic  $(1,1)$  supergravity in two dimensions, and show that this theory is equivalent to two scalar supermultiplets coupled to the usual Einstein supergravity. It is demonstrated that the theory possesses stable vacua with vanishing cosmological constant which spontaneously break supersymmetry.

## 1. BOSONIC GRAVITATION

The usual Einstein theory of gravitation involves a symmetric tensor  $g_{\mu\nu}$  whose dynamics is determined by the Lagrangian

$$\mathcal{L} = \frac{1}{2\kappa^2} \mathcal{R}. \quad (1)$$

The diffeomorphic gauge group reduces the number of degrees of freedom from ten down to six. Einstein's equations further reduce the degrees of freedom to two, which correspond to a physical spin-2 massless graviton. Now let us consider an extension of Einstein's theory by including terms in the action which are quadratic in the curvature tensors. This extended Lagrangian is given by

$$\mathcal{L} = \frac{1}{2\kappa^2} \mathcal{R} + \alpha \mathcal{R}^2 + \beta (C_{\mu\nu\alpha\beta})^2 + \gamma (\mathcal{R}_{\mu\nu})^2, \quad (2)$$

where  $\mathcal{R}^2$ ,  $(C_{\mu\nu\alpha\beta})^2$ , and  $(\mathcal{R}_{\mu\nu})^2$  are a complete set of CP-even quadratic curvature terms. The topological Gauss-Bonnet term is given by

$$\text{GB} = (C_{\mu\nu\alpha\beta})^2 - 2(\mathcal{R}_{\mu\nu})^2 + \frac{2}{3} \mathcal{R}^2. \quad (3)$$

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Therefore, we can write

$$\mathcal{L} = \frac{1}{2\kappa^2}\mathcal{R} + a\mathcal{R}^2 - b(C_{\mu\nu\alpha\beta})^2 + c \text{ GB}. \quad (4)$$

In this case, it can be shown [1] that there is still a physical spin-2 massless graviton in the spectrum. However, the addition of the  $\mathcal{R}^2$  term introduces a new physical spin-0 scalar,  $\phi$ , with mass  $m = (12a\kappa^2)^{-1/2}$ . Similarly, the  $(C_{\mu\nu\alpha\beta})^2$  term introduces a spin-2 symmetric tensor,  $\phi_{\mu\nu}$ , with mass  $m = (4b\kappa^2)^{-1/2}$  but this field, having wrong sign kinetic energy, is ghost-like. The GB term, being a total divergence, is purely topological and does not lead to any new degrees of freedom. The scalar  $\phi$  is perfectly physical and can lead to very interesting new physics [2]. The new tensor  $\phi_{\mu\nu}$ , however, appears to be problematical. There have been a number of attempts to show that the ghost-like behavior of  $\phi_{\mu\nu}$  is illusory, being an artifact of linearization [3]. Other authors have pointed out that since the mass of  $\phi_{\mu\nu}$  is near the Planck scale, other Planck scale physics may come in to correct the situation [4]. In all these attempts, the gravitational theories being discussed were not necessarily consistent and well defined. However, in recent years, superstring theories have emerged as finite, unitary theories of gravitation. Superstrings, therefore, are an ideal laboratory for exploring the issue of the ghost-like behavior of  $\phi_{\mu\nu}$ , as well as for asking whether the scalar  $\phi$  occurs in the superstring Lagrangian. Hence, we want to explore the question “*Do quadratic gravitation terms appear in the  $N = 1$ ,  $D = 4$  superstring Lagrangian?*”

Before doing this, however, we would like to present further details about the emergence of the new degrees of freedom in quadratic gravitation. We begin by adding to Einstein gravitation, quadratic terms associated with the scalar curvature only. That is, we consider the action

$$\mathcal{S} = \int d^4x \sqrt{-g} \left( \mathcal{R} + \frac{1}{6}m^{-2}\mathcal{R}^2 \right). \quad (5)$$

The equations of motion derived from this action are of fourth order and their physical meaning is somewhat obscure. These equations can be reduced to second order, and their physical content illuminated, by introducing an auxiliary field  $\phi$ . The action then becomes

$$\mathcal{S} = \int d^4x \sqrt{-g} \left( \mathcal{R} + \frac{1}{6}m^{-2}\mathcal{R}^2 - \frac{1}{6}m^{-2} [\mathcal{R} - 3m^2 \{e^\phi - 1\}]^2 \right) \quad (6)$$

$$= \int d^4x \sqrt{-g} \left( e^\phi \mathcal{R} - \frac{3}{2}m^2 [e^\phi - 1]^2 \right). \quad (7)$$

Note that the  $\phi$  equation of motion sets the square bracket in equation (6) to zero. Hence, action (7) with the auxiliary field  $\phi$  is equivalent to the original action (5). Now, let us perform a Weyl rescaling of the metric

$$g_{\mu\nu} = e^{-\phi} \bar{g}_{\mu\nu}. \quad (8)$$

It follows that

$$\begin{aligned}\sqrt{-g} &= e^{-2\phi} \sqrt{-\bar{g}}, \\ \mathcal{R} &= e^\phi \left( \bar{\mathcal{R}} + 3\bar{\nabla}^2 \phi - \frac{3}{2} [\bar{\nabla} \phi]^2 \right),\end{aligned}\tag{9}$$

where  $\bar{\nabla}_\lambda \bar{g}_{\mu\nu} = 0$ . Therefore,

$$\begin{aligned}\sqrt{-g} e^\phi \mathcal{R} &= \sqrt{-\bar{g}} \left( \bar{\mathcal{R}} + 3\bar{\nabla}^2 \phi - \frac{3}{2} [\bar{\nabla} \phi]^2 \right), \\ -\frac{3}{2} m^2 \sqrt{-g} (e^\phi - 1)^2 &= -\frac{3}{2} m^2 \sqrt{-\bar{g}} (1 - e^{-\phi})^2,\end{aligned}\tag{10}$$

and the action becomes

$$\mathcal{S} = \int d^4x \sqrt{-\bar{g}} \left( \bar{\mathcal{R}} - \frac{3}{2} [\bar{\nabla} \phi]^2 - \frac{3}{2} m^2 [1 - e^{-\phi}]^2 \right),\tag{11}$$

where we have dropped a total divergence term. It follows that the higher-derivative pure gravity theory described by action (5) is equivalent to a theory of normal Einstein gravity coupled to a real scalar field  $\phi$ . It is important to note that, with respect to the metric signature  $(-, +, +, +)$  we are using, the kinetic energy term for  $\phi$  has the correct sign and, hence, that  $\phi$  is not ghost like. Also, note that a unique potential energy function

$$V(\phi) = \frac{3}{2} m^2 (1 - e^{-\phi})^2\tag{12}$$

emerges which has a stable minimum at  $\phi = 0$ . We conclude that  $\mathcal{R} + \mathcal{R}^2$  gravitation with metric  $g_{\mu\nu}$  is equivalent to  $\bar{\mathcal{R}}$  gravitation with metric  $\bar{g}_{\mu\nu}$  plus a non-ghost real scalar field  $\phi$  with a fixed potential energy and a stable vacuum state. The property that  $\phi$  is non-ghost like is sufficiently important that we will present yet another proof of this fact. This proof was first presented in [2]. If we expand the metric tensor as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},\tag{13}$$

then the part of action (5) quadratic in  $h_{\mu\nu}$  is given by

$$\mathcal{S} = \int d^4x \left[ \frac{1}{4} h^{\mu\nu} \left( \nabla^2 \{ P_{\mu\nu\rho\sigma}^{(2)} - 2P_{\mu\nu\rho\sigma}^{(0)} \} + 2m^{-2} (\nabla^2)^2 P_{\mu\nu\rho\sigma}^{(0)} \right) h^{\rho\sigma} \right],\tag{14}$$

where  $P_{\mu\nu\rho\sigma}^{(2)}$  and  $P_{\mu\nu\rho\sigma}^{(0)}$  are transverse projection operators for  $h_{\mu\nu}$ . Inverting the kernel yields the propagator

$$\begin{aligned}\Delta_{\mu\nu\rho\sigma}^{-1} &= (\nabla^2 P_{\mu\nu\rho\sigma}^{(2)} + 2m^{-2} \nabla^2 [\nabla^2 - m^2] P_{\mu\nu\rho\sigma}^{(0)})^{-1} \\ &= \frac{1}{\nabla^2} \left( P_{\mu\nu\rho\sigma}^{(2)} - \frac{1}{2} P_{\mu\nu\rho\sigma}^{(0)} \right) + \frac{1}{2(\nabla^2 - m^2)} P_{\mu\nu\rho\sigma}^{(0)}.\end{aligned}\tag{15}$$

The term proportional to  $(\nabla^2)^{-1}$  corresponds to the usual two helicity massless graviton. However, the term proportional to  $(\nabla^2 - m^2)^{-1}$  represents the propagation of a real scalar

field with positive energy and, hence, not a ghost. This corresponds to the results obtained using the auxiliary field above. We would like to point out that there may be very interesting physics associated with the scalar field  $\phi$ . For example, as emphasized in [5],  $\phi$  may act as a natural inflaton in cosmology of the early universe.

Now let us consider Einstein gravity modified by quadratic terms involving the Weyl tensor only. That is, consider the action

$$\mathcal{S} = \int d^4x \sqrt{-g} \left( \mathcal{R} - \frac{1}{2} m^{-2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} \right). \quad (16)$$

Using the identity

$$C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} = \text{GB} + 2 \left( \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} - \frac{1}{3} \mathcal{R}^2 \right), \quad (17)$$

where GB is the topological Gauss-Bonnet combination defined in (3), the action becomes

$$\mathcal{S} = \int d^4x \sqrt{-g} \left( \mathcal{R} - m^{-2} \left[ \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} - \frac{1}{3} \mathcal{R}^2 \right] \right), \quad (18)$$

where we have dropped a total divergence. The fourth order equations of motion can be reduced to second order equations by introducing an auxiliary symmetric tensor field  $\phi_{\mu\nu}$ . Using this field, the action can be written as

$$\mathcal{S} = \int d^4x \sqrt{-g} \left( \mathcal{R} - G_{\mu\nu} \phi^{\mu\nu} + \frac{m^2}{4} [\phi_{\mu\nu} \phi^{\mu\nu} - \phi^2] \right), \quad (19)$$

where  $\phi = \phi_{\mu\nu} g^{\mu\nu}$  and  $G_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R}$  is the Einstein tensor. Note that the  $\phi_{\mu\nu}$  equation of motion is

$$\phi_{\mu\nu} = 2m^{-2} \left( \mathcal{R}_{\mu\nu} - \frac{1}{6} g_{\mu\nu} \mathcal{R} \right). \quad (20)$$

Substituting this into (19) gives back the original action (18). As it stands, action (19) is somewhat obscure since the  $G_{\mu\nu} \phi^{\mu\nu}$  term mixes  $g_{\mu\nu}$  and  $\phi_{\mu\nu}$  at the quadratic level. They can, however, be decoupled by a field redefinition. First write the above action as

$$\mathcal{S} = \int d^4x \sqrt{-g} \left( \left[ \left\{ 1 + \frac{1}{2} \phi \right\} g^{\mu\nu} - \phi^{\mu\nu} \right] \mathcal{R}_{\mu\nu} + \frac{m^2}{4} [\phi_{\mu\nu} \phi^{\mu\nu} - \phi^2] \right). \quad (21)$$

Now transform the metric as

$$\sqrt{-g} g^{\mu\nu} = \sqrt{-g} \left( \left[ 1 + \frac{1}{2} \phi \right] g^{\mu\nu} - \phi^\mu{}_\alpha g^{\alpha\nu} \right), \quad (22)$$

or, equivalently,

$$\begin{aligned} g_{\mu\nu} &= (\det A)^{-1/2} A_\mu{}^\alpha \bar{g}_{\alpha\nu}, \\ A_\mu{}^\alpha &= \left( 1 + \frac{1}{2} \phi \right) \delta_\mu{}^\alpha - \phi_\mu{}^\alpha. \end{aligned} \quad (23)$$

Under this transformation

$$\mathcal{R}_{\mu\nu} = \overline{\mathcal{R}}_{\mu\nu} - \overline{\nabla}_\mu C^\alpha_{\alpha\nu} + \overline{\nabla}_\alpha C^\alpha_{\mu\nu} + C^\alpha_{\mu\nu} C^\beta_{\alpha\beta} - C^\alpha_{\mu\beta} C^\beta_{\nu\alpha}, \quad (24)$$

where  $\overline{\nabla}_\lambda \overline{g}_{\mu\nu} = 0$  and

$$\begin{aligned} C^\alpha_{\mu\nu} &= \frac{1}{2} (X^{-1})^{\alpha\beta} (\overline{\nabla}_\mu X_{\nu\beta} + \overline{\nabla}_\nu X_{\mu\beta} - \overline{\nabla}_\beta X_{\mu\nu}), \\ X_{\mu\nu} &= g_{\mu\nu} = (\det A)^{-1/2} A_\mu{}^\alpha \overline{g}_{\alpha\nu}. \end{aligned} \quad (25)$$

Inserting these transformations into the above and dropping a total divergence, the action becomes [6]

$$\mathcal{S} = \int d^4x \sqrt{-\overline{g}} \left[ \overline{\mathcal{R}} + \overline{g}^{\mu\nu} (C^\alpha_{\mu\nu} C^\beta_{\beta\alpha} - C^\alpha_{\mu\beta} C^\beta_{\nu\alpha}) - \frac{m^2}{4} (\det A)^{-1/2} (\phi_{\mu\nu} \phi^{\mu\nu} - \phi^2) \right]. \quad (26)$$

Note that the action for  $\phi_{\mu\nu}$  is a complicated non-linear sigma model since  $C = C(X)$  and  $X = X(\phi)$ . It is useful to consider the kinetic energy part of the action expanded to quadratic order in  $\phi_{\mu\nu}$  only. It is found to be

$$\mathcal{S}_\phi^{\text{quad}} = \int d^4x \sqrt{-\overline{g}} \left( \frac{1}{4} \overline{\nabla}^\alpha \phi^{\mu\nu} \overline{\nabla}_\alpha \phi_{\mu\nu} - \frac{1}{2} \overline{\nabla}^\alpha \phi^{\mu\nu} \overline{\nabla}_\mu \phi_{\nu\alpha} + \frac{1}{2} \overline{\nabla}_\mu \phi^{\mu\nu} \overline{\nabla}_\nu \phi - \frac{1}{4} \overline{\nabla}^\alpha \phi \overline{\nabla}_\alpha \phi \right). \quad (27)$$

This action is clearly the curved space generalization of the Pauli-Fierz action for a spin-2 field except that every term has the wrong sign! This implies, of course, that  $\phi_{\mu\nu}$  propagates as a ghost. It is interesting to note that the action (26) is invariant under the gauge transformation

$$\begin{aligned} \phi'_{\mu\nu} &= \phi_{\mu\nu} + \overline{\nabla}_{(\mu} \xi_{\nu)} - C^\alpha_{\mu\nu} \xi_\alpha, \\ \overline{g}'_{\mu\nu} &= \left( 1 + \frac{1}{2} \phi' \right)^{-1} \left[ \frac{\det^{1/2} A'}{\det^{1/2} A} \left( 1 + \frac{1}{2} \phi \right) \overline{g}_{\mu\nu} + (\phi'_{\mu\nu} - \phi_{\mu\nu}) \right]. \end{aligned} \quad (28)$$

This insures that the above action describes a consistent coupling of a spin-2 symmetric tensor field  $\phi_{\mu\nu}$  to Einstein gravitation at the full non-linear level [7]. We conclude, therefore, that  $\mathcal{R} + C^2$  gravitation with metric  $g_{\mu\nu}$  is equivalent to  $\overline{\mathcal{R}}$  gravity with metric  $\overline{g}_{\mu\nu}$  plus a ghost-like symmetric tensor field  $\phi_{\mu\nu}$  with a consistent non-linear coupling to gravity and a fixed potential energy. The physics in the field  $\phi_{\mu\nu}$  is obscured by its ghost-like nature. However, its ghost nature can be altered by yet higher-derivative terms, such as those one would expect to find generated in superstring theories. Therefore, at long last, we turn to our discussion of quadratic supergravitation in superstring theory.

## 2. SUPERSPACE FORMALISM

In the Kähler (Einstein frame) superspace formalism, the most general Lagrangian for Einstein gravity, matter and gauge fields is

$$\mathcal{L}_E = -\frac{3}{2\kappa^2} \int d^4\theta E[K] + \frac{1}{8} \int d^4\theta \frac{E}{R} f(\Phi_i)_{ab} W^{\alpha a} W_{\alpha}{}^b + \text{h.c.}, \quad (29)$$

where we have ignored the superpotential term which is irrelevant for this discussion. The fundamental supergravity superfields are  $R$  and  $W_{\alpha\beta\gamma}$ , which are chiral, and  $G_{\alpha\dot{\alpha}}$ , which is Hermitian. The bosonic  $\mathcal{R}^2$ ,  $(C_{mnpq})^2$  and  $(\mathcal{R}_{mn})^2$  terms are contained in the highest components of the superfields  $\bar{R}R$ ,  $(W_{\alpha\beta\gamma})^2$  and  $(G_{\alpha\dot{\alpha}})^2$  respectively. One can also define the superGauss-Bonnet combination

$$\text{SGB} = 8(W_{\alpha\beta\gamma})^2 + (\bar{\mathcal{D}}^2 - 8R)(G_{\alpha\dot{\alpha}}^2 - 4\bar{R}R). \quad (30)$$

The bosonic Gauss-Bonnet term is contained in the highest chiral component of SGB. It follows that the most general quadratic supergravity Lagrangian is given by

$$\mathcal{L}_Q = \int d^4\theta E \left[ \Sigma(\bar{\Phi}_i, \Phi_i) \bar{R}R + \frac{1}{R} g(\Phi_i) (W_{\alpha\beta\gamma})^2 + \Delta(\bar{\Phi}_i, \Phi_i) (G_{\alpha\dot{\alpha}})^2 + \text{h.c.} \right]. \quad (31)$$

Although our discussion is perfectly general, we will limit ourselves to orbifolds, such as  $Z_4$ , which have  $(1,1)$  moduli only. The relevant superfields are the dilaton,  $S$ , the diagonal moduli  $T^{II}$ , which we'll denote as  $T^I$ , and all other moduli and matter superfields, which we denote collectively as  $\phi^i$ . The associated Kähler potential is

$$\begin{aligned} K &= K_0 + Z_{ij} \bar{\phi}^i \phi^j + \mathcal{O}((\bar{\phi}\phi)^2), \\ \kappa^2 K_0 &= -\ln(S + \bar{S}) - \sum (T^I + \bar{T}^I), \\ Z_{ij} &= \delta_{ij} \prod (T^I + \bar{T}^I)^{q_i^I}. \end{aligned} \quad (32)$$

The tree level coupling functions  $f_{ab}$  and  $g$  can be computed uniquely from amplitude computations and are given by

$$f_{ab} = \delta_{ab} k_a S, \quad g = S. \quad (33)$$

There is some ambiguity in the values of  $\Delta$  and  $\Sigma$  due to the ambiguity in the definition of the linear supermultiplet. We will take the conventional choice

$$\Delta = -S, \quad \Sigma = 4S. \quad (34)$$

It follows that, at tree level, the complete  $Z_N$  orbifold Lagrangian is given by  $\mathcal{L} = \mathcal{L}_E + \mathcal{L}_Q$  where

$$\mathcal{L}_Q = \frac{1}{4} \int d^4\theta \frac{E}{R} S \text{SGB} + \text{h.c.} \quad (35)$$

Using this Lagrangian, we now compute the one-loop moduli-gravity-gravity anomalous threshold correction [8]. This must actually be carried out in the conventional (string frame)

superspace formalism and then transformed to Kähler superspace [9]. We also compute the relevant superGreen-Schwarz graphs. Here we will simply present the result. We find that

$$\begin{aligned} \mathcal{L}_{\text{massless}}^{1\text{-loop}} = & \frac{1}{24(4\pi)^2} \sum \left[ h^I \int d^4\theta (\bar{\mathcal{D}}^2 - 8R) \bar{R} R \frac{1}{\partial^2} D^2 \ln(T^I + \bar{T}^I) \right. \\ & + (b^I - 8p^I) \int d^4\theta (W_{\alpha\beta\gamma})^2 \frac{1}{\partial^2} D^2 \ln(T^I + \bar{T}^I) \\ & \left. + p^I \int d^4\theta (8(W_{\alpha\beta\gamma})^2 + (\bar{\mathcal{D}}^2 - 8R)((G_{\alpha\dot{\alpha}})^2 - 4\bar{R}R)) \frac{1}{\partial^2} D^2 \ln(T^I + \bar{T}^I) + \text{h.c.} \right], \end{aligned} \quad (36)$$

where

$$\begin{aligned} h^I &= \frac{1}{12} (3\gamma_T + 3\vartheta_T q^I + \varphi), \\ b^I &= 21 + 1 + n_M^I - \dim G + \sum (1 + 2q_I^i) - 24\delta_{GS}^I, \\ p^I &= -\frac{3}{8} \dim G - \frac{1}{8} - \frac{1}{24} \sum 1 + \xi - 3\delta_{GS}^I. \end{aligned} \quad (37)$$

The coefficients  $\gamma_T$  and  $\vartheta_T$ , which arise from moduli loops, and  $\varphi$  and  $\xi$ , which arise from gravity and dilaton loops, are unknown. However, as we shall see, it is not necessary to know their values to accomplish our goal. Now note that if  $h^I \neq 0$  then there are non-vanishing  $\mathcal{R}^2$  terms in the superstring Lagrangian. If  $b^I - 8p^I \neq 0$  then the Lagrangian has  $C^2$  terms. Coefficient  $p^I \neq 0$  merely produces a Gauss-Bonnet term. With four unknown parameters what can we learn? The answer is, a great deal! Let us take the specific example of the  $Z_4$  orbifold. In this case, the Green-Schwarz coefficients are known [10]

$$\delta_{GS}^{1,2} = -30, \quad \delta_{GS}^3 = 0, \quad (38)$$

which gives the result

$$b^{1,2} = 0, \quad b^3 = 11 \times 24. \quad (39)$$

Now, let us try to set the coefficients of the  $(C_{\mu\nu\alpha\beta})^2$  terms to zero simultaneously. This implies that

$$b^I = 8p^I \quad (40)$$

for  $I = 1, 2, 3$  and therefore that

$$p^{1,2} = 0, \quad p^3 = 33. \quad (41)$$

From this one obtains two separate equations for the parameter  $\xi$  given by

$$\xi = \frac{3}{8} \dim G + \frac{1}{8} + \frac{1}{24} \sum 1 - 90, \quad (42)$$

for  $I = 1, 2$  and

$$\xi = \frac{3}{8} \dim G + \frac{1}{8} + \frac{1}{24} \sum 1, \quad (43)$$

for  $I = 3$ . Clearly these two equations are incompatible and, hence, it is impossible to have all vanishing  $(C_{\mu\nu\alpha\beta})^2$  terms in the 1-loop corrected Lagrangian of  $Z_4$  orbifolds. We find that the same results hold in other orbifolds as well.

### 3. SUPERSYMMETRY BREAKING IN $D = 2$ SUPERGRAVITY

Having demonstrated that quadratic gravitational terms can appear in four-dimensional,  $N = 1$  superstring Lagrangians, we would like to consider the effect of such terms on the vacuum state. Although our ultimate goal is to do this in superstring theory, at the present time we content ourselves with exploring the same issue in quadratic  $N = 1$  supergravity theories, which are simpler and less constrained. Here, we will present our results for  $D = 2$ ,  $(1, 1)$  supergravity. However, we have shown that similar results generically occur in  $D = 4$ ,  $N = 1$  supergravity theories as well. The two-dimensional,  $(1, 1)$  supergravity multiplet is composed of a graviton  $g_{mn}$ , a gravitino  $\chi_m^\alpha$  and a real auxiliary scalar field  $A$  [13]. The relevant superfields are the superdeterminant  $E$  given by

$$E = e \left( 1 + \frac{i}{2} \theta^\alpha \gamma^m_\alpha{}^\beta \chi_{m\beta} + \bar{\theta} \theta \left[ \frac{i}{4} A + \frac{1}{8} \tilde{\epsilon}^{mn} \chi_m^\alpha \gamma^5_\alpha{}^\beta \chi_{n\beta} \right] \right) \quad (44)$$

and a scalar superfield  $S$ , where

$$S = A + \theta^\alpha \psi_\alpha + \frac{i}{2} \bar{\theta} \theta C \quad (45)$$

and

$$\begin{aligned} C &= -\mathcal{R} - \frac{1}{2} \chi_m^\alpha \gamma^m_\alpha{}^\beta \psi_\beta + \frac{i}{4} \tilde{\epsilon}^{mn} \chi_m^\alpha \gamma^5_\alpha{}^\beta \chi_{n\beta} A - \frac{1}{2} A^2, \\ \psi_\alpha &= -2i \tilde{\epsilon}^{mn} \gamma^5_\alpha{}^\beta \mathcal{D}_m \chi_{n\beta} - \frac{i}{2} \gamma^m_\alpha{}^\beta \chi_{m\beta} A. \end{aligned} \quad (46)$$

The usual Einstein supergravity is described by the action

$$\mathcal{S}_E = 2i \int d^2x d^2\theta E S. \quad (47)$$

In component fields this simply becomes

$$\mathcal{S}_E = \int d^2x e \mathcal{R}, \quad (48)$$

which is a total divergence. That is, Einstein supergravity in two dimensions has no propagating degrees of freedom and is purely topological. The most general quadratic supergravity action is given by

$$\mathcal{S}_{E+Q} = 2i \int d^2x d^2\theta E (f(S) + g(S) \mathcal{D}^\alpha S \mathcal{D}_\alpha S), \quad (49)$$



where  $f$  and  $g$  are arbitrary functions of superfield  $S$ . Recall that in two dimensions the Weyl tensor vanishes, so this theory contains powers of the bosonic scalar curvature  $\mathcal{R}$  only. Furthermore, the structure of action (49) is such that all higher powers  $\mathcal{R}^n$  for  $n \geq 3$  vanish, and there are never more than two derivatives acting on the component field  $A$ . Here, for simplicity, we will consider the special case where

$$\begin{aligned} f(S) &= a + S + bS^2 + dS^3, \\ g(S) &= ic, \end{aligned} \tag{50}$$

and  $a, b, c$ , and  $d$  are real constants. In analogy with the bosonic case discussed in Section 1, we introduce two auxiliary scalar superfields,  $\Lambda$  and  $\Phi$ . The above Lagrangian is then equivalent to

$$\mathcal{L} = 2iE \left[ a + S + b\Lambda^2 + ic\mathcal{D}^\alpha \Lambda \mathcal{D}_\alpha \Lambda + d\Lambda^3 + (e^\Phi - 1)(S - \Lambda) \right]. \tag{51}$$

Inserting the equations of motion for  $\Lambda$  and  $\Phi$  back into this Lagrangian yields the original action (49). Again, in analogy with the bosonic case, we perform a superWeyl transformation of the form

$$\begin{aligned} \tilde{E} &= e^\Phi E, \\ \tilde{S} &= e^{-\Phi} S + ie^{-\Phi} \mathcal{D}^\alpha \mathcal{D}_\alpha \Phi. \end{aligned} \tag{52}$$

The Lagrangian then takes the form

$$\mathcal{L} = 2iE \left[ e^\Phi S + ie^\Phi \mathcal{D}^\alpha \Phi \mathcal{D}_\alpha \Phi + e^{-\Phi} (1 - e^\Phi) \Lambda + ae^{-\Phi} + be^{-\Phi} \Lambda^2 + ic\mathcal{D}^\alpha \Lambda \mathcal{D}_\alpha \Lambda + de^{-\Phi} \Lambda^3 \right], \tag{53}$$

where we have dropped the tilde. It follows that quadratic supergravity is equivalent to Einstein supergravity (modified by the  $e^\Phi$  factor in front of  $S$ ) coupled to two new scalar superfield degrees of freedom. Superfields  $\Lambda$  and  $\Phi$  can be expanded into component fields as

$$\begin{aligned} \Lambda &= \lambda + i\theta^\alpha \zeta_\alpha + \frac{i}{2} \bar{\theta} \theta G, \\ \Phi &= \phi + i\theta^\alpha \pi_\alpha + \frac{i}{2} \bar{\theta} \theta F. \end{aligned} \tag{54}$$

Inserting these expressions, and the expansion of  $S$ , into Lagrangian (53) and eliminating the auxiliary fields  $A$ ,  $G$  and  $F$  yields a component field Lagrangian of the form

$$\mathcal{L} = \mathcal{L}_{\text{KE-Boson}} - V(\phi, \lambda) + \mathcal{L}_{\text{KE-Fermion}} + \mathcal{L}_{\text{M-Fermion}} + \mathcal{L}_{\text{Boson-Fermion}}. \tag{55}$$

The boson kinetic energy term is given by

$$\mathcal{L}_{\text{KE-Boson}} = e^\phi \mathcal{R} - 2e^\phi (\nabla_m \phi)^2 - 2c (\nabla_m \lambda)^2. \tag{56}$$

Note that both  $\lambda$  and  $\phi$  are physically propagating scalar fields. The potential energy term is found to be

$$V = \frac{1}{8c} \left( 1 - 2e^{-\phi} [1 + 2b\lambda + (2c + 3d)\lambda^2] + e^{-2\phi} [1 + 4(b + ac)\lambda + 2(2b^2 + 2c + 3d)\lambda^2 + 4b(c + 3d)\lambda^3 + d(4c + 9d)\lambda^4] \right). \quad (57)$$

We now solve generically for extrema of this potential with vanishing cosmological constant. We find that all such extrema,  $(\lambda_0, \phi_0)$ , satisfy

$$\begin{aligned} a &= \frac{4}{27} \frac{b^3}{(c + 2d)^2}, \\ \lambda_0 &= \frac{-2b}{3(c + 2d)}, \\ 1 - e^{-\phi_0} &= \frac{-4b^2(c + 3d)}{9c^2 - 4b^2(c + 3d) + 36d(c + d)}. \end{aligned} \quad (58)$$

Evaluated at these extrema, the fermion mass term in the Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{\text{M-Fermion}} &= m_{11} i\pi^\alpha \pi_\alpha + m_{22} i\zeta^\alpha \zeta_\alpha + m_{33} \tilde{\epsilon}^{mn} \chi_m^\alpha \gamma_\alpha^5{}^\beta \chi_{n\beta} + 2m_{12} i\pi^\alpha \zeta_\alpha \\ &\quad + 2m_{13} i\pi^\alpha \gamma_\alpha^m{}^\beta \chi_{m\beta} + 2m_{23} i\zeta^\alpha \gamma_\alpha^m{}^\beta \chi_{m\beta}, \end{aligned} \quad (59)$$

where

$$\begin{aligned} m_{11} &= \frac{-2b(-2b^2 + 3c + 6d)}{P}, \\ m_{22} &= \frac{9bc(c + 2d)}{P}, \\ m_{33} &= \frac{2b^3c}{3(c + 2d)P}, \\ m_{12} &= \frac{-3c(3c - 4b^2 + 12d) + 12d(b^2 - 3d)}{2P}, \\ m_{13} &= \frac{bc(9c - 8b^2 + 36d) - 12bd(b^2 - 3d)}{6(c + 2d)P}, \\ m_{23} &= \frac{-2b^2c}{P}, \end{aligned} \quad (60)$$

where  $P$  is a polynomial in  $b, c$ , and  $d$  given by

$$P = c(9c - 4b^2 + 36d) - 12d(b^2 - 3d). \quad (61)$$

Some ranges of parameters  $b, c, d$  correspond to  $(\lambda_0, \phi_0)$  being a local maximum or a saddle point. However, for a large range of parameters, we find that  $(\lambda_0, \phi_0)$  is a local minimum. Furthermore, this minimum is very stable against quantum tunneling since the potential energy barrier around it is of the order of the Planck scale. Let us evaluate the fermion mass matrix for  $b, c$ , and  $d$  corresponding to such a minimum, and then diagonalize it. The result

is that, in a new fermion basis labeled by  $\tilde{\chi}_m^\alpha$ ,  $\tilde{\pi}$  and  $\tilde{\zeta}$ , the square of the fermion mass matrix is

$$\widetilde{M}^\dagger \widetilde{M} = \begin{pmatrix} m_{33}^2 & & \\ & 0 & \\ & & \tilde{m}^2 \end{pmatrix}, \quad (62)$$

where  $m_{33}$  and  $\tilde{m}$  are non-vanishing, and  $m_{33}$  is given in equation (60). Note the vanishing mass for  $\tilde{\pi}$ . This implies that  $\tilde{\pi}$  is a Goldstone fermion and, hence, that supersymmetry is spontaneously broken at this vacuum. This conclusion is further strengthened by the fact that the gravitino,  $\tilde{\chi}_m^\alpha$ , has acquired a non-vanishing mass. As a final check that supersymmetry is indeed broken, one can compute the supersymmetry transformation of the three diagonal fermions, evaluated at the vacuum. Schematically, we find that

$$\begin{aligned} \delta_{SUSY} \tilde{\chi}_m^\alpha &= \cdots + 0, \\ \delta_{SUSY} \tilde{\pi} &= \cdots + \text{non-zero}, \\ \delta_{SUSY} \tilde{\zeta} &= \cdots + 0. \end{aligned} \quad (63)$$

The inhomogeneous term in the supersymmetry transformation of  $\tilde{\pi}$ , proves that this vacuum spontaneously breaks supersymmetry and, in fact, is the reason why  $\tilde{\pi}$  is a massless Goldstone fermion.

We conclude that in two dimensional (1,1) quadratic supergravity there exist, for a large range of parameters, stable vacua with vanishing cosmological constant that spontaneously break the (1,1) supersymmetry. Supersymmetry is broken by two new scalar superfield degrees of freedom that are contained in the supervielbein in quadratic supergravity. Importantly, this result is not restricted to two dimensions. We have recently shown that exactly the same phenomenon occurs in  $D = 4$ ,  $N = 1$  quadratic supergravitation. It follows that higher derivative supergravity might serve as a natural mechanism for spontaneously breaking supersymmetry in phenomenologically interesting particle physics models. The results of our ongoing investigations will be presented elsewhere.

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